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Updating of Finite Element Models Using Vibration Tests

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Today the adjustment of structural models is an essential step in the modeling of complex structures. In this paper, we are interested in the improvement of finite element models. Our approach is a parametric updating using modal test results, which supply eigenvalues and associated eigenvectors. It is based on the computation of the error measure on the constitutive relation and allows us to correct both the stiffness and the mass matrices. In particular, this paper shows how this tuning strategy can improve a given finite element model when the measures are noisy. Several simulation examples illustrate the behavior of this method.

Nomenclature

A	= cross-sectional area
E	= Young's modulus
e	= strain operator
fi	= ith eigenfrequency of the finite element model
H	= Hooke's operator
I	= inertial moment
K_t	= stiffness matrix (tth tuning iteration)
K_0	= initial stiffness matrix
M_t	= mass matrix (ith tuning iteration)
M_0	= initial mass matrix
p_t^0	= updated design parameter (tth iteration of the correction stage)
r	= confidence scalar
X_i	= ith eigenvector of the finite element model
$(\Delta A/A)_i$	= relative error on the cross area (ith element)
$(\Delta I/I)_i$	= relative error on the inertial moment (ith element)
ε_i	= relative error measure for the whole structure for the ith experimental eigenshape
$\varepsilon_i(s)$	= relative error measure computed for the ith experimental eigenshape (sth substructure)
ε^q	= relative error measure computed for the whole structure and for all q measured modes
$\varepsilon^q(s)$	= relative error measure computed for all q measured eigenshapes (sth substructure)
$\eta^q(s)$	= error indicator computed for all q measured eigenshapes (sth substructure)
λ_i	= ith eigenvalue of the finite element model
$\underline{\lambda}_i$	= experimentally obtained ith eigenvalue
Π	= projection operator
ρ	= density
$\_\_$	= experimental (measured) values

Superscript

t	= transpose of a matrix or a vector

Introduction

THE problem of the control of a finite element model must be placed within the general framework of analysis-tests dialog. A recent tendency in engineering has been to reduce the number of tests and prefer numerical simulations; the tests are used to validate and verify the modeling. Such control problems appear for many free-vibration industrial problems where the results obtained with finite element models are not too far from the observed experimental results. Nevertheless, noticeable differences can be observed: these proceed from an erroneous estimation of the parameters describing the mass and stiffness properties. These errors are frequent in the modeling of joining substructures. When attempting to improve a finite element model, we are confronted with difficulties proceeding from the ill-posed character of this updating problem, mainly because of the limited number of sensors.

A key point of tuning methods consists in defining the error measure between the experimental values and the corresponding computed results. Direct methods as reported by Baruch,[1] Berman and Flannelly,[2] Chen and Garba,[3] He and Ewins,[5] Ewins and He,[6] and Link et al.[7] construct the corrected mass matrix and stiffness matrix, using the measured modal characteristics and orthogonality relations. Other methods are indirect methods, which consist in optimization approaches. Thus, some authors (Wei and Allemang,[8] Collins et al.,[9] Zhang et al.,[10] and Dascotte and Vanhonecker[11]) introduce sensitivity techniques to locate the most erroneous substructures first. Variants are proposed by Cottin et al.,[12] Niedbal et al.,[13] and Nash[14]; these try to improve the mass matrix, the damping matrix, and the stiffness matrix by minimizing the input errors, i.e., the difference between the computed forces and the experimental forces, or by minimizing the output errors, i.e., the difference between the experimental displacement and the analytical displacement.

Another key point for complex structures is that it is impossible to take all of the structural parameters of the finite element model into account at the same time. A reasonable prerequisite is to strictly limit the number of structural parameters involved. For that purpose, it is necessary to locate the erroneous zones of the structure first.

A limited number of methods place particular emphasis on the problem of errors localization. For example, the approach shown by Berger et al.[15,16] localizes the erroneous substructures by computing the residues of the equilibrium equations.

For our approach, the quality of a given finite element model is defined for all measured eigenmodes by a global error and by local errors relating to the substructures. These are errors on the constitutive relation. The zones where the local error is the highest are corrected as a priority. The tuning strategy uses an iterative process with each iteration containing a localization stage and a correction stage.

The specificity of our approach is that we take into account the good quality of the experimentally obtained eigenvalues. Then, to correct the finite element model, we try to find their associated eigenmodes. Nevertheless, the measured part of the experimental eigenmodes is introduced with a confidence coefficient. These eigenmodes do not need to be given in order. However, to include the orthogonality conditions, we order them according to a quality measure.

The principle of the localization method has been given by Ladeveze[17] and later developed by Ladeveze and Reynier[18] and by Reynier[19] with an associated correction process. In this paper, we

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describe the behavior of this tuning strategy when the tests results present noticeable noise effects. Both the stiffness matrix and the mass matrix characterizing the given finite element model are assumed to be erroneous.

Basic Approach: Error Measure

The data of the tuning problem are as follows: the finite element model; the q available experimental eigenvalues $\underline{\lambda}_i$, $i \in [1, q]$; and the measured part of the associated experimental eigenshapes $\underline{\Pi U}_i$, where Π is the projection operator indicating that the experimental eigenshape is partly measured.

Let Ω be a bounded subset with the boundary $\partial\Omega$ corresponding to the structure. To specify the boundary conditions let $\partial_1\Omega$ and $\partial_2\Omega$ be two complementary subsets. Consider U_d the displacement field given on $\partial_1\Omega$ and the normal stress vector given on $\partial_2\Omega$. Each experimental eigenvalue $\underline{\lambda}$ is considered as a right eigenvalue of the desired finite element model. We then attempt to construct each complete associated displacement by solving for each $\underline{\lambda}$ in the following problem:

Find a couple (U, σ) where U is a displacement field and σ a stress field such that U satisfies the kinematic constraints, $U \in \boldsymbol{U}$, where

$$\boldsymbol{U} = (U', U'|\partial_1\Omega = 0, U' \text{ regular}) \tag{1}$$

Such that (U, σ) satisfies the equilibrium equation

$$\forall U^* \in \boldsymbol{U}, \qquad \int_\Omega \mathrm{Tr}[\sigma e(U^*)]\, \mathrm{d}\Omega = \underline{\lambda} \int_\Omega \rho U \cdot U^*\, \mathrm{d}\Omega \tag{2}$$

and such that (U, σ) verifies the constitutive relation

$$\sigma = He(U) \tag{3}$$

Equations (1) and (2) mean (U, σ) is an admissible couple.

This problem is rewritten, introducing the error measure on the constitutive relation, and we associate to the $\underline{\lambda}$ eigenvalue the admissible couple (U, σ), which minimizes the error measure on the constitutive relation:

Find (U, σ) belonging to $\boldsymbol{A}_d$,

$$\boldsymbol{A}_d = [(U', \sigma'), U'|\partial_1\Omega = 0, U' \text{ regular, and } (U', \sigma') \text{ verifies Eq. (2)}]$$

such that they minimize

$$\boldsymbol{J}: (U', \sigma') \to \boldsymbol{J}(U', \sigma') = \|\sigma' - He(U')\|^2 \tag{4}$$

$$\|\sigma'\|^2 = \int_\Omega \mathrm{Tr}(\sigma' H^{-1} \sigma')\, \mathrm{d}\Omega \tag{5}$$

and we solve the following problem:

Find (U, σ) the admissible couple that minimizes the error measure on the constitutive relation on $\boldsymbol{A}_d$.

Remark: If $\underline{\lambda}$ is a right eigenvalue of the given finite element model, an admissible couple (U, σ) can be found such that the error measure on the constitutive relation is equal to zero. Then the associated experimental eigenshape and the reference eigenshape are identical.

Displacement Approach

To obtain a displacement approach, an equivalent strategy consists in introducing the couple (U, V) where V is the displacement field solution of the following elastic problem:

$$\forall U^* \in \boldsymbol{U}, \qquad \int_\Omega \mathrm{Tr}[He(V) - \sigma]\, e(U^*)\, \mathrm{d}\Omega = 0 \tag{6}$$

Finally, using the available measures $\underline{\Pi U}$, we solve the following problem where the measure of the global modified error on the constitutive relation is minimized:

Find $U \in \boldsymbol{U}$ and $V \in \boldsymbol{U}$ such that they minimize

$$\boldsymbol{E}^2: (U', V') \to \boldsymbol{E}^2(U', V') = \|U' - V'\|^2 + \frac{r}{1-r} |||\Pi U' \; \underline{\Pi U}|||^2 \tag{7}$$

with

$$\|U'\|^2 = \int_\Omega \mathrm{Tr}[He(U')e(U')]\, \mathrm{d}\Omega \tag{8}$$

and such that they verify the equilibrium constraint:

$$\forall U^* \in \boldsymbol{U}, \qquad \int_\Omega \mathrm{Tr}[He(V')e(U^*)]\, \mathrm{d}\Omega = \underline{\lambda} \int_\Omega \rho U' U^*\, \mathrm{d}\Omega \tag{9}$$

The quantity $\boldsymbol{E}(U, V)$ measures the quality of the couple (U, V) associated to the theoretical model and to the experimental mode $(\underline{\lambda}, \underline{\Pi U})$.

Remarks: In this paper, we do not present the way we take into account the modes' orthogonality, but these properties are systematically introduced as constraints by means of Lagrange multipliers. Before that, we order the experimental eigenmodes according to the quality measure defined by the normalized modified error measure on the constitutive relation.

For each mode shape i, the obtained displacement field U_i extends the experimental part $\underline{\Pi U}_i$.

The solution (0, 0) is dismissed when $\underline{\Pi U}$ is different from zero.

The $|||\ldots|||$ is an energy norm chosen on the truncated space where the part of the experimental shape is known. The choice of this norm is of minor importance as regards the localization quality.

The variable r is a scalar expressing the confidence in the quality of the experimental shapes; a current value is 1/2. For very noisy measures, low values should be chosen.

Local Errors Measures—Localization Method

For each given experimental mode shape $(\underline{\lambda}_i, \underline{\Pi U}_i)$, the model correctness is measured by means of the relative error measure on the constitutive relation computed for the whole structure and for all q measured modes:

$$\varepsilon^q = \left[\sum_{i=1}^{q} \frac{\|U_i - V_i\|^2}{(1/2)(\|U_i\|^2 + \|V_i\|^2)} \right]^{1/2} \tag{10}$$

In practice, if ε^q is lower than the test accuracy, the theoretical model is assumed to offer a good representation of the experimental behavior.

For a structure divided into substructures (s), the relative error computed for the experimental eigenshape i is given by

$$\varepsilon_i(s) = \left(\frac{\int_S \mathrm{Tr}\,[K\varepsilon(U_i - V_i)\varepsilon(U_i - V_i)]\, \mathrm{d}s}{1/2 \left\{ \int_\Omega \mathrm{Tr}\,[K\varepsilon(U_i)\varepsilon(U_i)]\, \mathrm{d}\Omega + \int_\Omega \mathrm{Tr}[K\varepsilon(V_i)\varepsilon(V_i)]\, \mathrm{d}\Omega \right\}} \right)^{1/2} \tag{11}$$

For the q experimental modes

$$\varepsilon^q(s) = \left\{ \sum_{i=1}^{q} [\varepsilon_i(s)]^2 \right\}^{1/2}$$

characterizes the local error measure computed for the q measured eigenshapes and for the substructure s. We also use the following indicator $\eta^q(s)$, which takes into account the energy levels of the substructures:

$$\eta^q(s) = \left[\sum_{i=1}^{q} \frac{\|U_i - V_i\|_S^2}{1/2(\|U_i\|_s^2 + \|V_i\|_s^2)}\right]^{1/2}$$

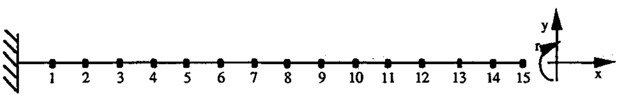
with

$$\|U\|_S^2 = \int_S \mathrm{Tr}\,[K\varepsilon(U)\varepsilon(U)]\,\mathrm{d}s$$

The most erroneous zones are associated to the maximal values of $\varepsilon^q(s)$ and $\eta^q(s)$. These localized areas should be corrected as a priority.

Updating Finite Element Models

For a free-vibration problem without damping, the finite element model is characterized by the symmetric mass matrix M_0 and the stiffness matrix K_0, the dimension n of which represents the number of degrees of freedom (DOF). The m first eigenvalues and their

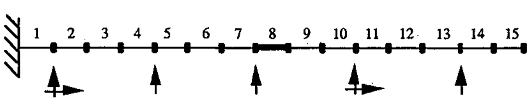


a) Initial Finite Element Model - 15 finite elements

bending and bar element - total length = 1.80 m

$A = (10)^{-4}\,\mathrm{m}^2$ $\quad I = 8.33\ (10)^{-10}\,\mathrm{m}^4$

$E = 2.\ (10)^{11}\,\mathrm{Pa}$ $\quad \rho = 7800\ \mathrm{kg/m}^3$

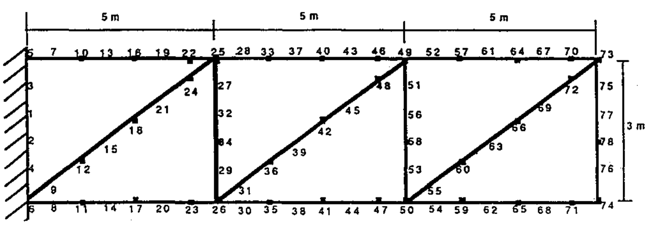


b) Simulated real structure

and measured degrees of freedom

Modified element : 8 (nodes 7-8), $(\Delta I/I)_8 = +100\%$

Fig. 1 First simulation example: two-dimensional cantilever beam.



a) Initial Finite Element Model

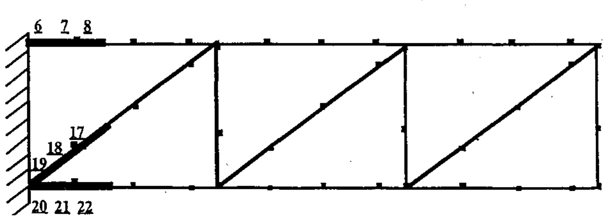
78 elements (bending and bar element) - 216 d.o.f.

$E = 0.75\ (10)^{11}$ Pa, $I = 0.0756\ \mathrm{m}^4$, $\rho = 2800\,\mathrm{kg/m}^3$

vertical elements : $A = 0.6\ (10)^{-2}\ \mathrm{m}^2$

diagonal elements : $A = 0.3\ (10)^{-2}\ \mathrm{m}^2$

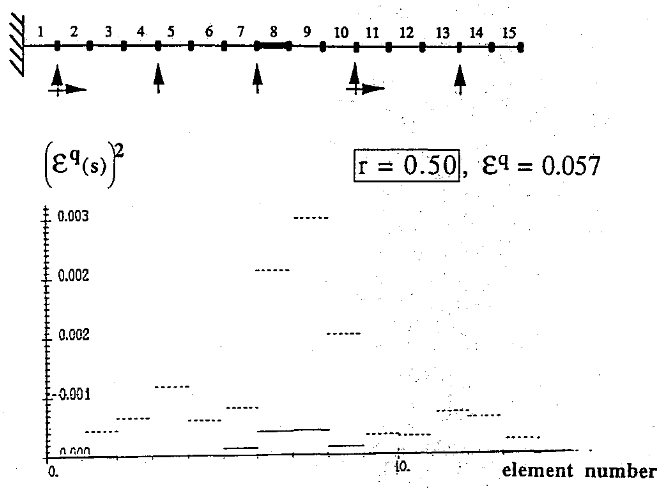
horizontal elements : $A = 0.4\ (10)^{-2}\ \mathrm{m}^2$



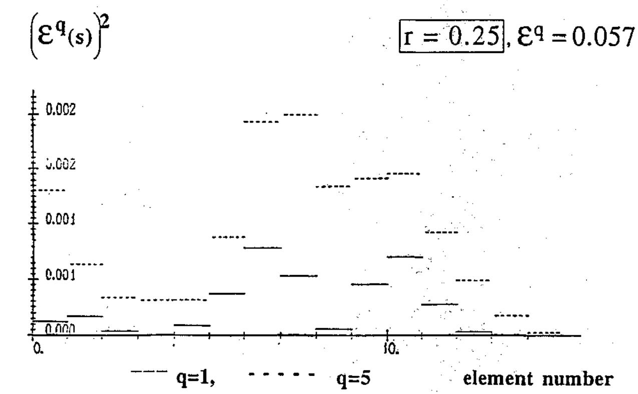
b) Simulated Real Structure and Sensor Location ▪

Modified elements 6 (nodes 5 - 7), 7 (7-10), 8 (10-13), 17 (nodes 12-15), 18 (9-12), 19 (6 -9), 20 (nodes 6 - 8), 21 (8-11), 22 (11-14).

Fig. 2 Third test case of the GARTEUR group: in-plane clamped-free vibrations of undamped truss system.



a) Localization stage (without noise)



b) Localization stage (with noise)

Fig. 3 Two-dimensional cantilever beam: localization results.

associated eigenmodes are computed ($m \ll n$). On the other hand, experimentally, we have q eigenvalues with r measured components of the associated eigenmodes ($q \ll m$) ($r \ll n$). We suppose that the r components are measured on the nodes of the finite element model and thus represent a part of the measured generalized displacement. Let u and v be the nodal values of U and V. We then obtain the following problem:

Find the displacement fields (u,v) minimizing

$$\underset{\sim}{E}^2\!: (u', v') \rightarrow \underset{\sim}{E}^2(u', v') = \|u' - v'\|^2 + \frac{r}{1-r}\,|||\pi u' \underline{\pi u}|||^2 \qquad (12)$$

with the constraint $K_0 v' = \underline{\lambda}\ M_0 u'$

$$u' \in \boldsymbol{u}, \qquad v' \in \boldsymbol{u} \qquad \text{where} \qquad \boldsymbol{u} = \{u', u'|\partial_1\Omega = 0\} \qquad (13)$$

Remarks: The choice of $|||\ldots|||^2$ is of minor importance: we use Guyan's reduction K_{r0} of the stiffness matrix K_0 on the measured DOFs.

The error measure can be formally expressed by means of the experimental information ($\underline{\lambda}$, $\underline{\pi u}$). The equilibrium constraint is written

$$u - v = Qu \qquad (14)$$

The solution u verifies

$$\left(Q^t K_0 Q + \frac{r}{1-r} K_{r0}\right) u = \frac{r}{1-r} K_{r0}\,\underline{\pi u} \qquad (15)$$

Consider

$$K_{r0} = \begin{bmatrix} K_r & 0 \\ 0 & 0 \end{bmatrix}$$

Using the following partition

$$Q^t K_0 Q = \begin{bmatrix} A & B \\ B^t & D \end{bmatrix}$$

writing

$$\underline{\pi u} = \begin{bmatrix} \pi u_r \\ 0 \end{bmatrix}, \qquad u = \begin{bmatrix} \pi u \\ (I - \pi)u \end{bmatrix}$$

and using Eq. (15),

$$e^2(u, v) = -\frac{r}{1-r}\,\underline{\pi u}_r{}^t\, K_{r0}(\pi u - \underline{\pi u}_r)$$

and leads to

$$e^2(u, v) = -\left(\frac{r}{1-r}\right)^2 \pi u_r{}^t K_r \left[(A - BD^{-1}B^t) + \frac{r}{1-r}\right]^{-1} K_r \underline{\pi u}_r$$

$$+ \frac{r}{1-r}\,\underline{\pi u}_r{}^t K_r \underline{\pi u}_r \tag{16}$$

If $\underline{\lambda}$ is the right eigenvalue for the proposed finite element model, we verify that Eq. (15) is written $(r/1-r)\,K_{r0}\,\pi u = (r/1-r)\,K_{r0}\,\underline{\pi u}$, and consequently $e^2(u, v) = -(r/1-r)\,\underline{\pi u}^t K_{r0}(\pi u - \underline{\pi u})$ is equal to 0.

Relation Between the Error Measure and the Errors on Structural Parameters

The global error measure is hereafter shown as being directly connected to the error on the stiffness matrix and on the mass matrix. We consider the case where the experimental eigenvector is completely given for the measured eigenmode i. We use a truncated modal base $[(Xk, \lambda_k), k \in (1, m)]$ of the given finite element to describe u and v, with the equilibrium equation supplying v:

$$u = \sum_{k=1}^{m} a_k X_k \qquad v = \sum_{k=1}^{m} \frac{\lambda_i}{\lambda_k} a_k X_k \tag{17}$$

Table 1 Simulated modifications

Element	6	7	8	17	18	19	20	21	22
$\Delta A/A$, %	100	100	100	0	0	0	0	0	0
$\Delta I/I$, %	25	25	0	−80	−80	−83.3	−83.3	−83.3	−83.3

Table 2 Comparison between initial analyzed and experimental eigenfrequencies

Mode no.	1	2	3	4	5
$\Delta f_i/f_i$, %	−20.3	0.7	−3.25	−11.85	0.25

Table 3 Results of the first correction stage[a]

$\Delta I_{18} = -0.6064\ (10)^{-1}$ m^4
$\Delta I_{19} = -0.5343\ (10)^{-1}$ m^4
$\Delta I_{20} = -0.6372\ (10)^{-1}$ m^4
$\Delta I_{21} = -0.5836\ (10)^{-1}$ m^4
$\Delta I_{22} = -0.6735\ (10)^{-1}$ m^4

[a]The subscript indicates the number of the corrected element.

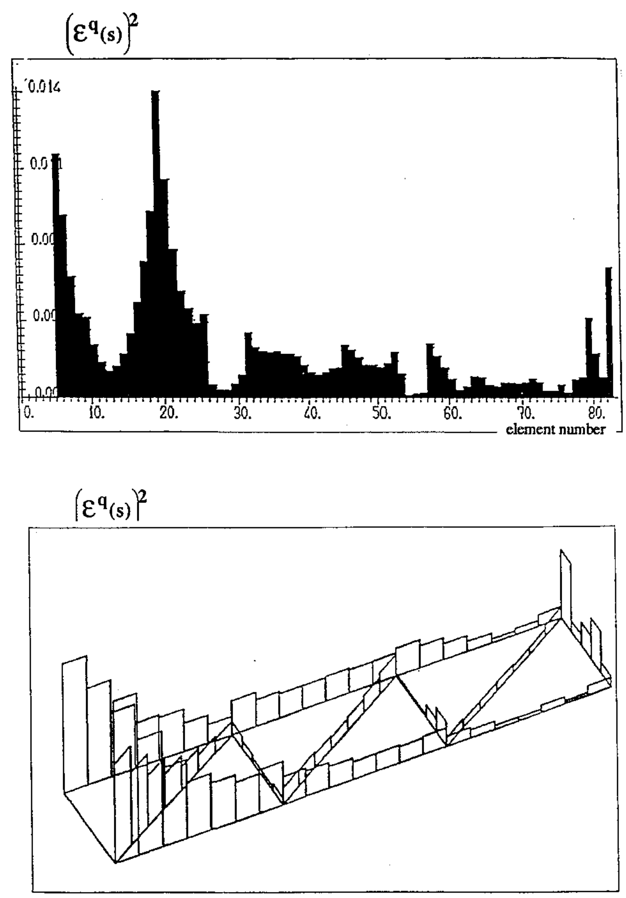


Fig. 4 First localization stage: $r = 0.1$, $\varepsilon^q = 0.127$, and $q = 10$.

For the experimental mode i, we obtain, using $X_i{}^t K X_j = \delta ij$ Kronecker,

$$\varepsilon_i \# \left[\left(\frac{\Delta\lambda_i}{\lambda_i}\right)^2 a_i{}^2 + \sum_{k=1}^{m}\left(\frac{\lambda_i}{\lambda_k} - 1\right)^2 a_k{}^2\right]^{1/2}, \qquad i \neq k \tag{18}$$

with $a_i \# 1$ and

$$a_k \# \frac{r}{1-r}\frac{X_k{}^t K \Delta X_i}{[(\underline{\lambda}_i/\lambda_k) - 1]^2 + (r/1-r)} \tag{19}$$

The classical sensitivity approach gives the X_i and λ_i variations:

$$(\Delta\lambda_i/\lambda_i) = X_i{}^t(\Delta K - \lambda_i \Delta M)X_i$$

$$\Delta X_i = \sum_{k=1}^{m} A_{ki} X_k \tag{20}$$

$$A_{ki} = \frac{X_k{}^t(\Delta K - \lambda_i \Delta M)X_i}{\lambda_k - \lambda_i} x_i$$

Then ε_i can be expressed using Eqs. (18–20):

$$(\varepsilon_i)^2 \# [X_i{}^t(\Delta K - \lambda_i \Delta M)X_i]^2 + \sum_{k=1, i\neq k}^{m}\left(\frac{\lambda_i}{\lambda_k} - 1\right)^2$$

$$\times \left[\frac{r}{1-r}\,\frac{X_k{}^t K \sum_{j=1}^{m}\left(\dfrac{X_j{}^t(\Delta K - \lambda_i \Delta M)X_i}{\lambda_k - \lambda_j} Xj\right) Xk}{\left(\dfrac{\lambda_i}{\lambda_k} - 1\right)^2 + \dfrac{r}{1-r}}\right]^2$$

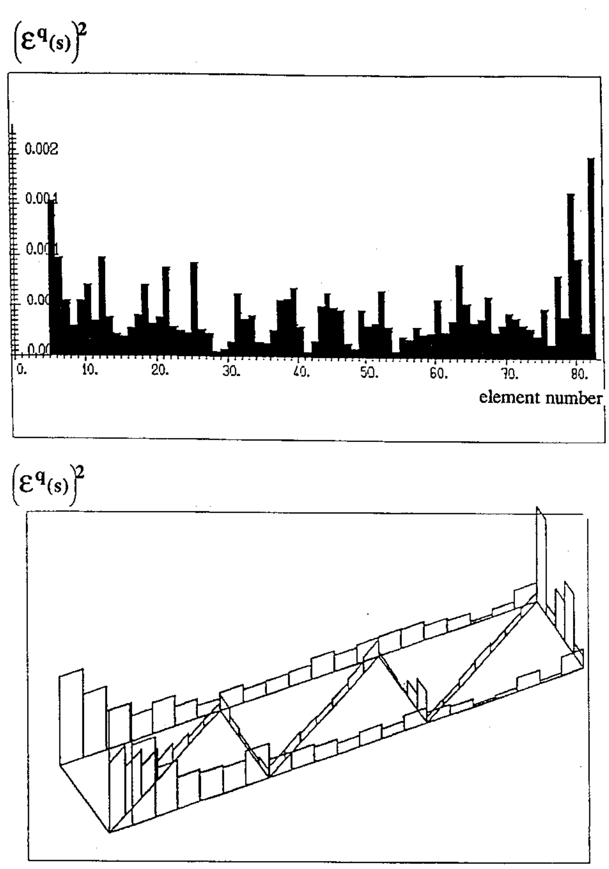


Fig. 5 Second localization stage: $r = 0.1$, $\varepsilon^q = 0.054$, and $q = 10$.

Correction Process

As a priority, the values of the structural parameters p_t (tth iteration) relating to the recognized erroneous areas are introduced in the expression of the global error measure for the whole structure and for the q given experimental modes.

The stiffness matrix is expressed by $K_t = K_{t-1} + \Delta K(p_t)$ and the mass matrix by $M_t = M_{t-1} + \Delta M(p_t)$. Then the correction problem is written for the iteration t as follows:

Find $p_t \in P_t$ minimizing

$$e^q\colon\ p' \to e^q(p') = \left[\sum_{i=1}^{q}\left(\|u_i - v_i\|^2 + \frac{r}{1-r}\|\pi u_i - \underline{\pi u}_i\|^2\right)\right]^{1/2}$$

$\boldsymbol{P}_t = (p_t$, such that they insure the properties of K_t and M_t) with $K_t v_i = \underline{\lambda}_i M_t u i$, $i \in (1, q)$.

Taking the equilibrium equations into account, the previous problem becomes the following one:

Find ($u_i \in \boldsymbol{U}$, $p_t \in \boldsymbol{P}_t$) such that they minimize

$$F\colon\ u', p' \to F(u_i', p')$$

$$= \left\{\sum_{i=1}^{q}\left[\|Q(p')u_i'\|^2 + \frac{r}{1-r}\|\pi u_i' + \underline{\pi u}_i\|^2\right]\right\}^{1/2}$$

Consequently,

$$F(u_i, p') = \sum_{k=1}^{q}\left[\left(\frac{r}{1-r}\right)(\underline{\pi u}_i)^t K\,(\pi u_i - \pi u_i)\right]$$

and finally we have to solve the following problem:

Find p_t minimizing on $\boldsymbol{P}_t$:

$$H\colon\ p' \to H(p') = F(u_i', p')$$

For each iteration t, the correction problem is nonlinear, but the number of variables is very low. We use a conjugate gradient algorithm that needs less than 10 iterations to compute the structural corrections p_t for the studied examples.

Remark: To describe the displacements fields, the numerical implementation uses a reduced base that is initially a truncated modal base. It is modified for each iteration t using the new matrices M_t and K_t.

Examples

To show the quality of the localization process, we propose two examples of updating problems using a clamped-free cantilever beam and a clamped-free plane truss structure. These simulated cases are depicted on Figs. 1 and 2. The structures are discretized into sample beams with lumped mass distribution. The real structures are simulated by modifying the geometrical parameters, and the modal parameters are recomputed with a finite element program. Inaccuracies have been numerically introduced to simulate noise effects. The analytical data of the simulated test model are perturbed: the frequencies with 2–3% noise, the components of the associated eigenvectors with 10% (see Ref. 21). The modifications on the structure concern elements in localized areas.

In. Fig. 1, a two-dimensional cantilever beam with a cross-sectional area $A = (10)^{-4}$ m^2 and an inertial moment $I = 8.33\ (10)^{-10}$ m^4 is discretized using 15 beam elements (bending and bar element). The finite element model contains 45 DOF (30 translational and 15 rotational), of which only 7 translational DOF are assumed to be measured. To simulate a modeling error, the inertial moment of member 8 is increased and set to $I_8 = 16.66\ (10)^{-10}$ m^4. Figure 3 shows that eigenvalues with 2% noise and eigenvectors with 10% noise do not distort the quality of the localization too much. Our strategy enables us to locate the element number 8, and the first tuning iteration gives a satisfactory correction equal to $\Delta I = 6.65\ (10)^{-10}$ m^4 for noised experimental information and equal to $\Delta I = 6.05\ (10)^{-10}$ m^4 for measured values without noise.

The second updating problem describes the third benchmark (Fig. 2) of the Group for Aeronautical Research and Technology in Europe (GARTEUR).[21] The "test" structure is a plane clamped-

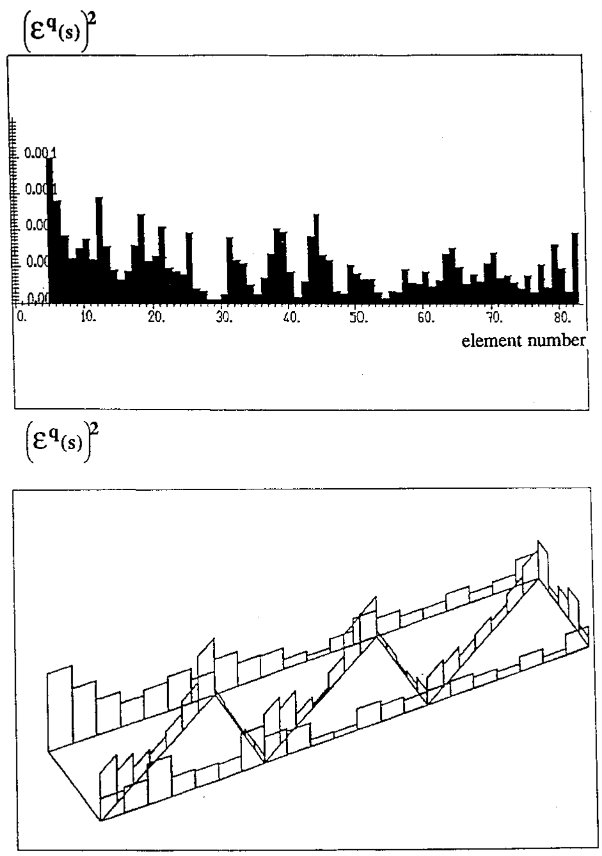
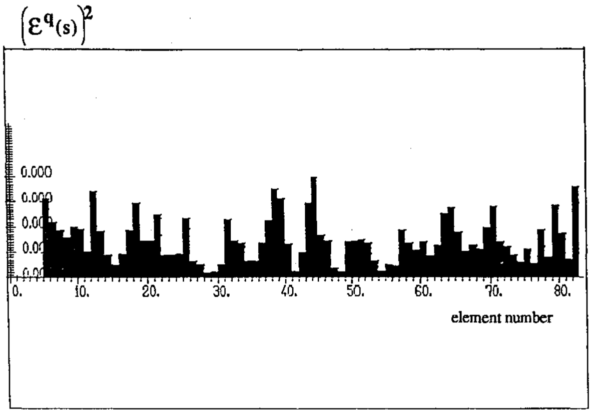


Fig. 6 Second localization stage: $r = 0.05$, $\varepsilon^q = 0.043$, and $q = 10$.



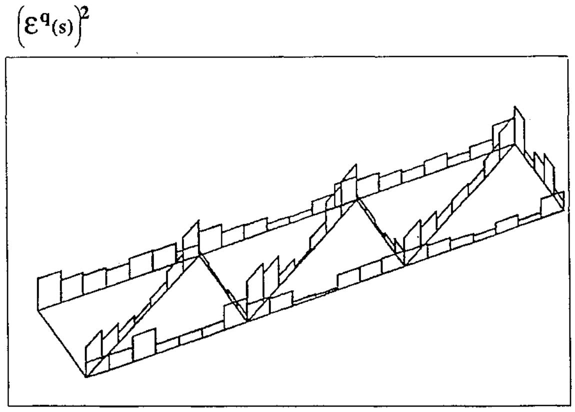


Fig. 7 Third and final localization stage: $r = 0.05$, $\varepsilon^q = 0.037$, and $q = 10$.

Table 4 Results of the second correction stage

$\Delta I_6 = +0.162\ (10)^{-1}\ m^4$
$\Delta I_7 = +0.213\ (10)^{-1}\ m^4$
$\Delta A_6 = +0.364\ (10)^{-2}\ m^2$
$\Delta A_7 = +0.301\ (10)^{-2}\ m^2$

Table 5 Final corrections of the structural parameters

Element	6	7	18	19	20	21	22
$I_{simulated}$	1.25*I	1.25*I	0.20*I	0.167*I	0.167*I	0.167*I	0.167*I
$I_{corrected}$	1.21*I	1.28*I	0.20*I	0.290*I	0.157*I	0.228*I	0.110*I
$A_{simulated}$	2.00*A	2.00*A					
$A_{corrected}$	1.91*A	1.75*A					

Table 6 Comparison between final analyzed and experimental eigenfrequencies

Mode no.	1	2	3	4	5
$\Delta f_i/f_i$, %	1.55	2.55	−0.30	−1.80	0.15

free truss system, whose members are characterized by $E = 0.75\ (10)^{11}$ Pa, $I = 0.0756\ m^4$, and $\rho = 2800\ kg/m^3$, with $A_{\text{vertical elements}} = 0.6\ (10)^{-2}\ m^2$, $A_{\text{diagonal elements}} = 0.3\ (10)^{-2}\ m^2$, and $A_{\text{horizontal elements}} = 0.4\ (10)^{-2}\ m^2$. The initial finite element model contains 216 DOF, of which 78 translational DOF are assumed to be measured. To simulate modeling errors, the modifications are listed in Table 1. The difference between the analyzed and the experimental (simulated) eigenvalues are described in Table 2 (for the first five modes).

The first localization stage (Fig. 4) allows us to locate elements 6, 7, 18, 19, 20, 21, and 22. The global error measure value is 0.127 computed with $r = 0.1$. Consequently, the model needs improvement, and the correction computing supplies the modifications listed in Table 3 after two iterations of the conjugate gradient algorithm.

The second localization stage (Fig. 5) supplies a global error equal to 0.0545 with $r = 0.1$. Proceeding further in the correction process with $r = 0.1$, we obtain too noisy an error map to localize new errors. If we want to improve the model further, we decrease r to give less importance to the experimental mode shapes as regards the experimental frequencies. For the value $r = 0.05$ we localize elements 6 and 7 (Fig. 6). The global error measure becomes 0.043. The computed corrections are given in Table 4.

Finally, the third localization stage (Fig. 7) gives a global error equal to 0.037 with $r = 0.05$. The final corrections are given in Table 5. The difference between the corrected and the experimental (simulated) eigenvalues for the first 5 modes are given in Table 6.

Conclusion

Our strategy uses the error measure on the constitutive relation, assuming that the structural parameters are the most erroneous where the error indicators are the highest. This tuning process remains satisfactory when confronted with noised experimental data. We are planning future developments relating to this method, in particular concerning the optimal location of sensors.

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